Speed Estimates From Slide To Stop Formula

I have noticed a large number of inquires on this website seeking information about speed estimates from the "slide to stop" formula. So this is my technical derivation and explanation.

The "Slide to Stop" formula is used widely by police and other investigators to place an estimate on how much speed was lost from decelerating over a certain distance. This commonly occurs when an investigator examines a motor vehicle accident and observes a set of skid marks.

The derivation of this formula is as follows:

Kinetic Energy is dissipated through Work. Thus Kinetic Energy equals the Work done while travelling over a distance.

Kinetic Energy is defined as 1/2 Mass x (Velocity)squared.

Work is defined as Force x Distance.

By equating the two we get:

1/2 Mass x (Velocity)squared = Force x Distance

We can replace the "Force" term by its components. The vehicle's weight which is really its Mass times the acceleration due to gravity, or "mg". We also include a variable, u, which is dependent on the quality of interaction between the tire and road surface. Thus we get:

Force = u x m x g, or Force = umg

In other words, the Force is a product of the co-efficient of friction (u), Mass, and the acceleration due to gravity, g. This makes sense because, as we skid along a road surface the contact between the tire and road surface produces a force that occurs because the earth's gravitational pull is pressing the car down onto the pavement. If there was gravitation pull there would be no "weight" and there would be no force. The car's motion sustains resistance because of the interface between the tire and pavement. This resistance is quantified by a number, often between 0 and 1, which is the co-efficient of friction, that changes depending on the qualities of the tire rubber, pavement characteristics, environmental conditions and other smaller influences. The other terms of the Force are the Mass (m) and the earth's gravitation pull, or "g". Thus the equation can be re-written as follows:

1/2 x M x (V)squared = umgd

So the terms on the left refer to the Kinetic Energy and the terms on the right refer to the Work (FxD) which has been re-written to show the components of the Force term.

The "M" term on the left is the same as the "m" term on the right because they both refer the Mass. So they can be cancelled out. Now we get:

1/2 x (V)squared = u x g x d

Now we simply move the terms on the left side of the "=" sign over to the right while keeping the "V" term. So first we move the "1/2" across the right side of the "=" sign. The "1/2" is simply "2" when we cross the equal sign. So we get:

(V)squared = 2 x u x g x d, or (V)squared = 2ugd

Now we can replace the "g" term with its actual value which, in the metric system, is 9.81 metres per second per second. In other words, an object in free fall increases it velocity at the rate of 9.81 metres per second in every second that it is falling. So now we have:

(V)squared = 2 x u x 9.81 x d

We can multiply the "2" by the "9.81" to simplify the terms so now we get:

(V)squared = 19.62 x u x d

We can now take the square root of both sides to leave the "V" term on the left on its own, resulting in:

V = Sqrt (19.62 x u x d)

The formulation can take on several different appearances depending on how we finalize it. Presently the velocity result will be in terms of meters per second. This is not always easily understood so that we often want the value in terms of "kilometres per hour". To transpose we need to multiple both sides by 3.6 because this the factor by which we transfer between "metres per second" and "kilometres per hour". We now refer to the result as "Speed" simply to imply that we are talking in terms of kilometres per hour. Thus we get:

3.6 x V = 3.6 x Sqrt (19.62 x u x d), or

Speed = 3.6 x Sqrt (19.62 x u x d)

Now we can bring the "3.6" under the square root sign by multiplying the terms by 3.6 x 3.6 or 12.96, thus we get:

Speed = Sqrt (12.96 x 19.62 x u x d)

To simplify we multiply the "12.96" by "19.62" and get:

Speed = Sqrt (254.28 x u x d)

The "254.82" is often seen rounded off to "255" thus the final equation often looks like:

Speed = Sqrt (255ud)

This means that the speed is dependent on the co-efficient of friction and distance that the vehicle has travelled while skidding, or decelerating. The deceleration is more technically correct as a "negative acceleration" but this can be confusing to the lay folk so we use the word "deceleration" for ease of understanding.

This equation is a simplification but it applies in a large number of cases without considering other factors. However, often the "u" is adjusted and referred to as a "drag factor", depending on one's definitions. Thus when the skidding or decelerating occurs on a non-level surface such as a downgrade or upgrade the "u" needs to be altered. In that case persons refer to it as a "drag factor" to indicate that the value is not just dependent on the mechanical property of friction between two surfaces.

Thus it could be possible for such a drag factor to be above the normal maximum of "1.0" which is equivalent to "g". For example, with maximum braking on an aggressive road surface where the normal co-efficient of friction is found to be .95, if the vehicle is sliding up a 10 % slope then the factor would be increased to 1.05 and referred to as a "drag factor". This naming is simply by convention in the reconstruction industry.

Similarly, on a slippery road surface covered by ice it is possible to have a very low co-efficient of friction, such as 0.5, and while travelling onto a steep downgrade of 10%, you would actually see an acceleration or increase in the vehicle's speed even though the brakes were applied to a maximum.

The slide to stop formulation is popular because it is simple to use. One only needs to think of a number between 0 and 1 where a value near 0 would indicate a very slippery surface and a value near 1 would indicate a very aggressive surface. Then we only need to consider how far the vehicle moved during the braking, skidding or deceleration.

While simple to use it is the reason why so many invalid and inaccurate results are obtained resulting in convictions or lack of convictions, huge liability payments in civil litigation, and so on. It requires someone who knows the methods of accident reconstruction, is honest, and is willing to question things before leaping to a conclusion. Using the formula on its own, because it is simple to obtain a quick answer, is simply a quick path to disaster.

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